

# Computational notes on the Numerical Analysis of Galactic Rotation Curves

G. Scelza\*, <sup>1</sup>A. Stabile†

<sup>1</sup>*Dipartimento di Ingegneria, Università del Sannio,  
Palazzo Dell'Aquila Bosco Lucarelli, Corso Garibaldi, 107 - 82100, Benevento, Italy*

In this paper we present a brief discussion on the salient points of the computational analysis that are at the basis of the paper [1]. The computational and data analysis have been made with the software Mathematica<sup>®</sup> and presented at Mathematica Italia User Group Meeting 2011.

PACS numbers: XX

Keywords: XX

## I. INTRODUCTION

The computational analysis here described is referred to the study of the galactic rotation curve. The theoretical details of the model investigated are omitted here, but fully available on the cited paper [1]. The formula under study is  $v(r, R, z) = \sqrt{r \frac{\partial}{\partial r} \Phi(r, R, z)}$  where  $\Phi(r, R, z)$  is the gravitational potential

$$\begin{aligned} \Phi(r, R, z) = & \frac{4\pi G}{3} \left[ \frac{1}{r} \int_0^\infty dr' \rho_{bulge}(r') r' \left( 3 \frac{|r-r'| - r - r'}{2} - \frac{e^{-\mu_1|r-r'|} - e^{-\mu_1(r+r')}}{2\mu_1} + \right. \right. \\ & \left. \left. + 2 \frac{e^{-\mu_2|r-r'|} - e^{-\mu_2(r+r')}}{\mu_2} \right) \right] + \\ & + \frac{4\pi G}{3} \left[ \frac{1}{r} \int_0^\Xi dr' \rho_{DM}(r') r' \left( 3 \frac{|r-r'| - r - r'}{2} - \frac{e^{-\mu_1|r-r'|} - e^{-\mu_1(r+r')}}{2\mu_1} + \right. \right. \\ & \left. \left. + 2 \frac{e^{-\mu_2|r-r'|} - e^{-\mu_2(r+r')}}{\mu_2} \right) \right] + \\ & - 2G \left[ \int_0^\infty dR' \sigma_{disc}(R') R' \left( \frac{\mathfrak{R}(\frac{4RR'}{(R+R')^2+z^2})}{\sqrt{(R+R')^2+z^2}} + \frac{\mathfrak{R}(\frac{-4RR'}{(R-R')^2+z^2})}{\sqrt{(R-R')^2+z^2}} \right) + \right. \\ & \left. + \int_0^\infty dR' \sigma_{disc}(R') R' \int_0^\pi d\theta' \frac{e^{-\mu_1\sqrt{(R+R')^2+z^2-4RR'\cos^2\theta'}} - 4e^{-\mu_2\sqrt{(R+R')^2+z^2-4RR'\cos^2\theta'}}}{3\sqrt{(R+R')^2+z^2-4RR'\cos^2\theta'}} \right] \end{aligned} \quad (1)$$

where  $\mathfrak{R}(x)$  is the Elliptic function and  $G$  is gravitational constant. We remember that in the potential (1) we can distinguish the contributions of the bulge, the disk and the (eventual) Dark Matter.  $r$  is the radial coordinate in the spherical system, while  $R, z$  are respectively the radial coordinate in the plane of disc and the distance from the plane then we have the geometric relation  $r = \sqrt{R^2 + z^2}$ . The main item is the choice of models of matter distribution. The more simple model characterizing the shape of galaxy is the following

$$\begin{cases} \rho_{bulge}(r) = \frac{M_b}{2\pi \xi_b^{3-\gamma} \Gamma(\frac{3-\gamma}{2})} \frac{e^{-\frac{r^2}{\xi_b^2}}}{r^\gamma} \\ \sigma_{disk}(R) = \frac{M_d}{2\pi \xi_d^2} e^{-\frac{R}{\xi_d}} \\ \rho_{DM}(r) = \frac{\alpha M_{DM}}{\pi (4-\pi) \xi_{DM}^3} \frac{1}{1 + \frac{r^2}{\xi_{DM}^2}} \end{cases} \quad (2)$$

\* e - mail address: lucasce73@gmail.com

† e - mail address: arturo.stabile@gmail.com

where  $\Gamma(x)$  is the Gamma function,  $0 \leq \gamma < 3$  is a free parameter and  $0 \leq \alpha < 1$  is the ratio of Dark Matter inside the sphere with radius  $\xi_{DM}$  with respect the total Dark Matter  $M_{DM}$ . Moreover the couples  $\xi_b$ ,  $M_b$  and  $\xi_d$ ,  $M_d$  are the radius and the mass of the bulge and the disc. The parameters  $\mu_1$  and  $\mu_2$  are the free parameters in the theory and only by fitting process can be fixed. A sensible item is the choice of distance  $\Xi$  on the which we are observing the rotation curve. In fact all models for the Dark Matter component are not limited and we need to cut the upper value of integration in (1).

A further distinction are the contributions to the potential coming from terms of General Relativity (GR) origin and terms of Forth Order Gravity (FOG) origin. Finally our aim is the numerical evaluation of the rotation curve in the galactic plane

$$v(R, R, 0) = \sqrt{R \frac{\partial}{\partial R} \Phi(R, R, 0)} \quad (3)$$

Our analysis is then organized as follows: in section II we investigate the contribution of these terms on the galactic rotation curve, in section III a data fit between our theoretical curves and the data of the rotation curve of the Milky Way and the galaxy NGC 3190 and in section IV we report the conclusions.

## II. THE COMPUTATION

The first step, after the definition of the numerical values for the parameters, has been the building of the velocity starting from de derivative of the potential as we can see in fig 1. The derivative and integration operations commute, then we “transport” the derivative in the the integrand and then we make the integration. We found this computationally more rapid. Moreover, we make a splitting in the GR contributions and FOG contributions in the gravitational potential. Then it follows the turning off the warning messages concerning the numerical integrations as showed in fig 2. Indeed for the first `Off`, as we can see in fig. 3, all the definitions are made with the “SetDelayed” command that postpones the numerical evaluation of the integral making it not immediately numerical. The following warnings inform us of the need to increase the precision of the computation. An interesting thing to note in fig 3, is that in the definition of the derivative by means of mute variables, it need not a “SetDelayed” command, but a simple “=” command.

$$\begin{aligned} \rho_b[r_-] &:= 0.43 \times e^{-\left(\frac{r}{a}\right)^2} (*\text{bulge density}*) \\ \rho_{DM}[r_-] &:= 0.4 \times \frac{1}{1 + \left(\frac{r}{\xi_{DM}}\right)^2} (*\text{Dark Matter density}*) \\ \sigma_d[y_-] &:= 0.8 \times e^{-\frac{y}{a}} (*\text{disk density}*) \\ \text{TerGr}[x_-, y_-] &:= 3 \frac{\text{ab}[x - y] - x - y}{2} \\ \text{TerYu}[x_-, y_-] &:= - \frac{e^{-\mu_1 \text{ab}[x-y]} - e^{-\mu_1 (x+y)}}{2 \mu_1} + 2 \frac{e^{-\mu_2 \text{ab}[x-y]} - e^{-\mu_2 (x+y)}}{\mu_2} \\ \text{F1}[r_-, rp_-] &:= \frac{\text{EllipticK}\left[\frac{4 r rp}{(r+rp)^2 + z^2}\right]}{\sqrt{(r+rp)^2 + z^2}} + \frac{\text{EllipticK}\left[-\frac{4 r rp}{(r-rp)^2 + z^2}\right]}{\sqrt{(r-rp)^2 + z^2}} \\ \text{F2}[r_-, rp_-, \theta p_-] &:= \frac{e^{-\mu_1 \sqrt{(r+rp)^2 - 4 r rp \cos[\theta p]^2 + z^2}} - 4 e^{-\mu_2 \sqrt{(r+rp)^2 - 4 r rp \cos[\theta p]^2 + z^2}}}{3 \sqrt{(r+rp)^2 - 4 r rp \cos[\theta p]^2 + z^2}} \end{aligned}$$

FIG. 1: The definition of the density terms and the splitting of GR contributions and FOG contributions to the gravitational potential

```
Off[NIntegrate::inumr]
Off[NIntegrate::slwcon]
Off[NIntegrate::ncvb]
Off[NIntegrate::eincr]
```

FIG. 2: Shooting off the warning messages

```

integrandbFOG[r_, rp_] :=  $\frac{2}{3 \xi b^{2-\gamma} \Gamma\left[\frac{3-\gamma}{2}\right]} \frac{1}{r} \rho b[rp] rp^{1-\gamma} (\text{TerGr}[r, rp] + \text{TerYu}[r, rp])$ 
DerintegrandbFOG[r_, rp_] = D[integrandbFOG[r, rp], r]; (*Interesting*)
bbFOG[R_] := NIntegrate[DerintegrandbFOG[r, rp], {rp, 0, 100  $\xi b$ },
    MaxRecursion -> 20, Method -> "GlobalAdaptive"] /. r ->  $\sqrt{R^2 + z^2}$ 
FOG[r_] := bbFOG[r] + dbFOG[r] + DMFOG[r]
VelFOG[r_] := K (r FOG[r])1/2
dataVelFOG = Table[VelFOG[R], {R, 10-7,  $\chi$ , stepr]];
fig2 = ListPlot[dataVelFOG, PlotRange -> All, FrameLabel ->
    {Style["R(Kpc)", Large, Black], Style["vc(R) (Km/s)", Large, Italic, Black]},
    DataRange -> {0,  $\chi$ }, PlotStyle -> Directive[Black, Thick],
    Joined -> True, AxesOrigin -> {0, 0}, Frame -> True]
th = Show[fig1, fig2, fig3, fig4]

```

FIG. 3: The derivative of the bulge term in the potential; case FOG

### III. DATA FIT

The next and more interesting step, is the comparison of the experimental data and what predicted by our model. From the literature cited in [1] we can obtain the galactic speed values as function of the distance from the center and the corresponding errors. For instance, we show in some detail the manipulation of the data coming from the analysis of [2], concerning the external part of the Milky Way. We start copying the data listed in the table 1 of [2] in

```

R0 = 10;  $\omega_0 = 220 / R_0$ ;  $\sigma_1 = 0.05$ ;
R = ( $R_0^2 + \text{list1}[[\text{All}, 3]]^2 - 2 R_0 \text{list1}[[\text{All}, 3]] \text{Cos}[\text{list1}[[\text{All}, 1]] \circ]$ )1/2;
list2 = MapThread[Append, {list1, R}];
 $\omega = \frac{\text{list2}[[\text{All}, 5]]}{R_0 \text{Sin}[\text{list2}[[\text{All}, 1]] \circ] \text{Cos}[\text{list2}[[\text{All}, 2]] \circ]} +$ 
 $\omega_0 - \frac{4.2 \text{Cos}[\text{list2}[[\text{All}, 1]] \circ]}{R_0 \text{Sin}[\text{list2}[[\text{All}, 1]] \circ]}$ ;
list3 = MapThread[Append, {list2,  $\omega$ }];
 $\sigma\omega = \text{Abs}\left[(\omega - \omega_0) \sqrt{\left(\frac{\text{list3}[[\text{All}, 6]]}{\text{list3}[[\text{All}, 5]]}\right)^2 + \left(\frac{\sigma_1}{\text{Tan}[\text{list3}[[\text{All}, 1]] \circ]}\right)^2}\right]$ ;
list4 = MapThread[Append, {list3,  $\sigma\omega$ }];
 $\theta = R \times \omega$ ;
list5 = MapThread[Append, {list4,  $\theta$ }];
 $\sigma R = \text{Abs}\left[\frac{1}{R} \sqrt{((\text{list4}[[\text{All}, 3]] - R_0 \text{Cos}[\text{list4}[[\text{All}, 1]] \circ])^2 \text{list4}[[\text{All}, 4]]^2 +}\right.$ 
 $\left. (R_0 \text{list4}[[\text{All}, 3]] \text{Sin}[\text{list4}[[\text{All}, 1]] \circ])^2 \sigma_1^2}\right]$ ;
list6 = MapThread[Append, {list5,  $\sigma R$ }];
list7 = Drop[list6, {}, {1, 6}];
 $\sigma\theta = \text{Abs}\left[\theta \sqrt{\left(\frac{\sigma\omega}{\omega}\right)^2 + \left(\frac{\sigma R}{R}\right)^2}\right]$ ;
list8 = MapThread[Append, {list7,  $\sigma\theta$ }];
list9 = Drop[list8, {}, {2, 3}];

```

FIG. 4: Data manipulation

a table called **list1**. Then we follow the prescriptions given by the authors with the introduction of new variables. As it is possible to see in fig. 4 we preserve the same notations and append to the initial **list1** the new variables. For instance, for the  $R$  variable, with the command **MapThread[Append, {list1, R}]**, we obtain a new table, here **list2** with one more column, the  $R$ 's value. And so on with the other variables. We computed, with the usual procedures, the errors on these derived quantities, here written as  $\sigma x$ . Then  $\sigma R$  and  $\sigma\theta$  are, respectively, the error bars on the radius (the distance from the galactic center) and on the corresponding speeds and we process them together with the data so as shown in the fig.5. In figure 6 it is shown the result. With the command in the second line of fig.5, we obtain a list whose elements are of kind **ErrorBar[err\_x, err\_y]**. In the third line we build a list whose element are **{x,y}, ErrorBar[err\_x, err\_y]}**. In these conditions, we need to load the package **ErrorBarPlot** in order to make

```

data = Drop[list9, {}, {3, 4}]
errx = Drop[Drop[list9, {}, {1, 2}], {}, {2}][[All, 1]]
erry = Drop[Drop[list9, {}, {1, 2}], {}, {1}][[All, 1]]
listerr = {errx, erry}^T
error = Cases[listerr, {x_, y_} >=> ErrorBar[x, y]]
valerr = Map[{#[[1]], #[[2]]} &, {data, error}]^T
Needs["ErrorBarPlots`"]
ErrorListPlot[valerr, FrameLabel ->
  {Style["R(Kpc)", Large, Black], Style["v_c(R) (Km/s)", Large, Italic, Black]},
  PlotStyle -> Black, PlotMarkers -> {"■"}, PlotRange -> All,
  AxesOrigin -> {0, 0}, Frame -> True]

```

FIG. 5: ErrorListPlot procedure

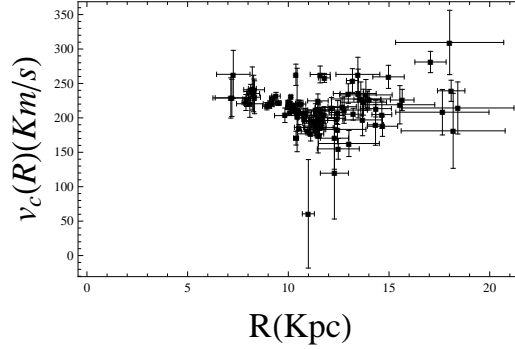


FIG. 6: ErrorListPlot

an `ErrorListPlot`. Similar procedures for the others two part of the Milky Way data and for the NGC 3190 data. At this point we proceeded following two strategies.

The first one, the faster, has been to overlap the theoretical graphs with the experimental one using the command `Show`. In this case, the values of parameters in the densities (bulge, Dark Matter and disk) and of reduced masses,  $\mu_1$  and  $\mu_2$ , see screen shot in fig1, are chosen by a direct overlap of the graphs.

The second strategy, more rigorous and slower, is the fit procedure. In this case, we fix all other parameters except the “masses”  $\mu_1$  and  $\mu_2$ . These variables are the values that must be found in the find fit procedure. We note that the

```

g1[R_,  $\mu_1$ _,  $\mu_2$ _] := (gb1[R,  $\mu_1$ ,  $\mu_2$ ] + gd1[R,  $\mu_1$ ,  $\mu_2$ ] + gDM1[R,  $\mu_1$ ,  $\mu_2$ ])
VelFOGDM[R_,  $\mu_1$ _,  $\mu_2$ _] := K (R g1[R,  $\mu_1$ ,  $\mu_2$ ])1/2
model = VelFOGDM[R,  $\mu_1$ ,  $\mu_2$ ];
FindFit[val, model, {{ $\mu_1$ , 10-2}, { $\mu_2$ , 102}}, R]

```

FIG. 7: FindFit procedure. The “masses”  $\mu_1$  and  $\mu_2$  are found by the fit with the experimental data, here represented by *val*

`FindFit` procedure uses the parameter constraints option. In this way, it is possible to eliminate all the solutions not physically allowed and to find the values obtained by the direct investigation, that is the first strategy,  $\mu_1 = 10^{-2}$ ,  $\mu_2 = 10^2$ .

#### IV. CONCLUSIONS

In this paper we presented the salient points in the program we build in the computation of the velocity curves of the Milky Way and the galaxy NGC 3190. In Fig. 8 is shown the full code corresponding to Fig. 6 of [1], that is the code for a galaxy whose components are the bulge, the disk and the Dark Matter. The code referring to the study of the galaxy NGC 3190 is exactly the same with the exclusion of the part of code referring to the bulge. As it is possible to see from Fig. 8 of [1], the agreement of our model with the experimental data of the Milky Way is very good. Only for very low values of the distance  $R$  the agreement is not perfect. This suggest us that we only need an improvement of the parameters in the code, maintaining the code itself essentially unchanged.

```

Off[NIntegrate::inumr]
Off[NIntegrate::slwcon]
Off[NIntegrate::ncvbj]
Off[NIntegrate::einocr]
Gr = UnitConvert[Quantity[1, "GravitationalConstant"]][[1]]; MSun = 1.98 × 1030;
μ1 = 10-2; μ2 = 102; Mb = 1.8 (*bulge mass*); Md = 6.5 (*disk mass*); MDM = 4.2
(*Dark Matter mass*); ξb = 0.5 (*bulge radius*); ξd = 3.5 (*galaxy radius*);
ξDM = 5.5 (*D.M. radius*); χ = 20; S = χ; stepr = 0.5; z = 5 × 10-5;
ab := H1 (HeavisideTheta[H1] - HeavisideTheta[-H1]) &

K = 10-3  $\sqrt{\frac{Gr \times 10^{10} \times MSun \times Mb}{10^5 \times 3.08 \times 10^{16} \times \xi b}}$ ;

ρb[r_] := 0.43 × e- $\frac{r}{\xi b}$ } (*bulge density*)
ρDM[r_] := 0.4 ×  $\frac{1}{1 + \left(\frac{r}{\xi DM}\right)^2}$  (*Dark Matter density*)
od[y_] := 0.8 × e- $\frac{y}{\xi d}$ } (*disk density*)
TerGr[x_, y_] := 3  $\frac{ab[x-y] - x - y}{2}$ 
TerYu[x_, y_] := -  $\frac{e^{-\mu_1 ab[x-y]} - e^{-\mu_1 (x+y)}}{2 \mu_1} + 2 \frac{e^{-\mu_2 ab[x-y]} - e^{-\mu_2 (x+y)}}{\mu_2}$ 
F1[x_, rp_] :=  $\frac{\text{EllipticK}\left[\frac{4 x rp}{(x+rp)^2 + z^2}\right]}{\sqrt{(x+rp)^2 + z^2}} + \frac{\text{EllipticK}\left[-\frac{4 x rp}{(x-rp)^2 + z^2}\right]}{\sqrt{(x-rp)^2 + z^2}}$ 
F2[x_, rp_, θp_] :=  $\frac{e^{-\mu_1 \sqrt{(x+rp)^2 - 4 x rp \cos(\theta p) + z^2}} - 4 e^{-\mu_2 \sqrt{(x+rp)^2 - 4 x rp \cos(\theta p) + z^2}}}{3 \sqrt{(x+rp)^2 - 4 x rp \cos(\theta p) + z^2}}$ 
integrandb1[x_, rp_, y_] :=  $\frac{2}{3 \xi b^{3-\gamma} \Gamma\left[\frac{3-\gamma}{2}\right]} \frac{1}{r} \rho b[rp] r p^{3-\gamma} (\text{TerGr}[r, rp] + \text{TerYu}[r, rp])$ 
Derintegrandb1[x_, rp_, y_] = D[integrandb1[x, rp, y], r];
$b1[R_, y_] := NIntegrate[Derintegrandb1[r, rp, y], {rp, 0, 100 ξb},
  MaxRecursion → 20, Method → "GlobalAdaptive"] /. r →  $\sqrt{R^2 + z^2}$ 
integrandDM1[x_, rp_, α_] :=  $\frac{4 \alpha MDM / Mb}{3 (4 - \pi) \xi DM^3 / \xi b r} \frac{1}{r} \rho DM[rp] r p (\text{TerGr}[r, rp] + \text{TerYu}[r, rp])$ 
DerintegrandDM1[x_, rp_, α_] = D[integrandDM1[x, rp, α], r];
$DM1[R_, α_] := NIntegrate[DerintegrandDM1[r, rp, α],
  {rp, 0, S}, MaxRecursion → 20, Method → "GlobalAdaptive"] /. r →  $\sqrt{R^2 + z^2}$ 
integrandd1[x_, rp_] := -  $\frac{(Md / Mb)}{\pi \frac{\xi d^2}{\xi b}} \times (rp \text{od}[rp] F1[r, rp])$ 
integrandd12[x_, rp_, θp_] := -  $\frac{(Md / Mb)}{\pi \frac{\xi d^2}{\xi b}} \times (rp \text{od}[rp] F2[r, rp, \theta p])$ 
Derintegrandd1[x_, rp_] = D[integrandd1[x, rp], r];
Derintegrandd12[x_, rp_, θp_] = D[integrandd12[x, rp, θp], r];
$d1[R_] := (NIntegrate[Derintegrandd1[r, rp], {rp, 0, 50 ξd}, MaxRecursion → 20,
  Method → "GlobalAdaptive"] + NIntegrate[Derintegrandd12[r, rp, θp],
  {rp, 0, 50 ξd}, {θp, 0, π}, MaxRecursion → 20, Method → "GlobalAdaptive"])
integrandb2[x_, rp_, y_] :=  $\frac{2}{3 \xi b^{3-\gamma} \Gamma\left[\frac{3-\gamma}{2}\right]} \frac{1}{r} \rho b[rp] r p^{3-\gamma} \text{TerGr}[r, rp]$ 
Derintegrandb2[x_, rp_, y_] = D[integrandb2[x, rp, y], r];
$b2[R_, y_] := NIntegrate[Derintegrandb2[r, rp, y], {rp, 0, 100 ξb},
  MaxRecursion → 20, Method → "GlobalAdaptive"] /. r →  $\sqrt{R^2 + z^2}$ 
integrandDM2[x_, rp_, α_] :=  $\frac{4 \alpha MDM / Mb}{3 (4 - \pi) \xi DM^3 / \xi b r} \frac{1}{r} \rho DM[rp] r p \text{TerGr}[r, rp]$ 
DerintegrandDM2[x_, rp_, α_] = D[integrandDM2[x, rp, α], r];
$DM2[R_, α_] := NIntegrate[DerintegrandDM2[r, rp, α],
  {rp, 0, S}, MaxRecursion → 20, Method → "GlobalAdaptive"] /. r →  $\sqrt{R^2 + z^2}$ 
(*integrandd2[x_, rp_] := -  $\frac{(Md / Mb)}{\pi \frac{\xi d^2}{\xi b}} \times rp \text{od}[rp] F1[r, rp]$  *)
Derintegrandd2[x_, rp_] = D[integrandd1[x, rp], r];
$d2[R_] := NIntegrate[Derintegrandd2[r, rp],
  {rp, 0, 50 ξd}, MaxRecursion → 20, Method → "GlobalAdaptive"]
$[x_, y_] := $b1[x, y] + $d1[x]
$[x_, y_] := $b2[x, y] + $d2[x]
$1[x_, y_, α_] := $[x, y] + $DM1[x, α]
$1[x_, y_, α_] := $[x, y] + $DM2[x, α]
VelFOG[x_, y_] := K (r $[x, y]) $\frac{1}{2}$ 
VelGR[x_, y_] := K (r $[x, y]) $\frac{1}{2}$ 
VelFOGDM[x_, y_, α_] := K (r $1[x, y, α]) $\frac{1}{2}$ 
VelGRDM[x_, y_, α_] := K (r $1[x, y, α]) $\frac{1}{2}$ 
dataVelFOG[y_] := Table[VelFOG[R, y], {R, 10-7, χ, stepr}];
dataVelGR[y_] := Table[VelGR[R, y], {R, 10-7, χ, stepr}];
dataVelFOGDM[y_, α_] := Table[VelFOGDM[R, y, α], {R, 10-7, χ, stepr}];
dataVelGRDM[y_, α_] := Table[VelGRDM[R, y, α], {R, 10-7, χ, stepr}];
fig1[y_] := ListPlot[dataVelGR[y], PlotRange → All, FrameLabel →
  {Style["R (Kpc)", Large, Black], Style["vc(R) (Km/s)", Large, Italic, Black]},
  DataRange → {0, χ}, PlotStyle → Directive[Black, Dashed, Thick],
  Joined → True, AxesOrigin → {0, 0}, Frame → True]
fig2[y_] := ListPlot[dataVelFOG[y], PlotRange → All, FrameLabel →
  {Style["R (Kpc)", Large, Black], Style["vc(R) (Km/s)", Large, Italic, Black]},
  DataRange → {0, χ}, PlotStyle → Directive[Black, Thick],
  Joined → True, AxesOrigin → {0, 0}, Frame → True]
fig3[y_, α_] := ListPlot[dataVelGRDM[y, α], PlotRange → All, FrameLabel →
  {Style["R (Kpc)", Large, Black], Style["vc(R) (Km/s)", Large, Italic, Black]},
  DataRange → {0, χ}, PlotStyle → Directive[Black, Dotted, Thick],
  Joined → True, AxesOrigin → {0, 0}, Frame → True]
fig4[y_, α_] := ListPlot[dataVelFOGDM[y, α], PlotRange → All, FrameLabel →
  {Style["R (Kpc)", Large, Black], Style["vc(R) (Km/s)", Large, Italic, Black]},
  DataRange → {0, χ}, PlotStyle → Directive[Black, Dotted, Thick],
  Joined → True, AxesOrigin → {0, 0}, Frame → True]
α = 0.5; γ = 1.5;
th = Show[fig1[y], fig2[y], fig3[y, α], fig4[y, α]]

```

FIG. 8: Screen-shot of the full program for the rotation curve of the Milky Way (Fig. 6 of [1]). A complete description of the plotted curves is present in [1]

- 
- [1] A.Stabile, G. Scelza, Phys. Rev. D **84**, 124023 (2011)
  - [2] M. Fich, L.Blitz, A.A. Stark, Ap. J. **342**, 272 (1989)

```

ρb[r_, a_] := a × e−( $\frac{r}{sb}$ )2 (*bulge density*)

ρDM[r_, b_] := b ×  $\frac{1}{1 + (\frac{r}{sDM})^2}$  (*Dark Matter density*)
model = VelFOG[R, y, a, b, c];
nlmf = NonlinearModelFit[val, model, {a, b, c}, R, PrecisionGoal → 40];
Print["Best Fit Parameters:"];

```